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Persistence in the zero-temperature dynamics of the diluted Ising ferromagnet in two dimensions

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The nonequilibrium dynamics of the strongly diluted random-bond Ising model in two dimensions (2D) is investigated numerically. The persistence probability, $P(t)$, of spins which do not flip by time t is found to decay to a nonzero, dilution-dependent value $P(\infty)$. We find that $p(t)=P(t)-P(\infty)$ decays *exponentially* to zero at large times. Furthermore, the fraction of spins which *never* flip is a monotonically increasing function over the range of bond-dilution considered. Our findings, which are consistent with a recent result of Newman and Stein [Phys. Rev. Lett. **82**, 3944 (1999)], suggest that persistence in diluted and pure systems falls into different classes. Furthermore, its behavior would also appear to depend crucially on the strength of the dilution present. [S1063-651X(99)50609-0]

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In the nonequilibrium dynamics of spin systems at zero-temperature we are interested in the fraction of spins, $P(t)$, that persist in the same state up to some later time t. For homogeneous ferromagnetic Ising models in d dimensions, $P(t)$, has been found to decay algebraically [1–4],

$$P(t)\sim t^{-\theta(d)}, \tag{1}$$

for $d<4$, where $\theta(d)$is the new nontrivial persistence exponent.

The presence of a nonvanishing $P(t)$ as $t\to\infty$ has been reported in computer simulations of both the Ising model in higher dimensions ($d>4$) [3] and the q-state Potts model in 2D for $q>4$ [5]; this feature is sometimes referred to as ''blocking.'' Obviously, if $P(\infty)>0$, we can reformulate the problem by restricting our attention only to those spins that eventually flip. Hence, we can consider the behavior of

$$p(t)=P(t)-P(\infty). \tag{2}$$

Although the numerical simulations of the q-state Potts model mentioned above [5] seem to indicate that $p(t)$also decays algebraically, the evidence is by no means conclusive. By considering the dynamics of the local order parameter, the persistence problem can be generalized to nonzero temperatures [6–9].

It is only recently [10,11] that attention has turned to the persistence problem in systems containing disorder. Numerical simulations of the zero-temperature dynamics of the weakly diluted Ising model in 2D [10] also reported that $P(\infty)>0$. In fact, the study in [10] is consistent with the presence of three distinct regimes: an initial short time regime where the behavior is purelike; an intermediate regime where the persistence probability decays logarithmically; and a final long time regime where the system ''freezes'' and $P(t)$ is effectively constant.

Very recently, Newman and Stein [11] have argued that the ''blocking'' [5] of spins in systems with continuous disorder is associated with the fact that ''every spin flips only finitely many times.'' As a consequence, in some simple 1D models $p(t)$ was found to decay exponentially rather than algebraically for large times, namely,

$$p(t)\sim e^{-kt}, \tag{3}$$

where $k>0$. In contrast, persistence in the weakly diluted Ising model appears to decay logarithmically in the intermediate regime. [Note that [10] examines the behavior of $P(t)$ and not $p(t)$].

Clearly, it is of immense interest to establish whether the presence of ''blocking'' in a system necessarily implies exponential decay of the persistence probability. Howard [12] has found evidence for exponential decay in certain nondisorderd models with ''blocking'' (2D hexagonal lattices and Bethe lattices with $z=3$ are discussed in [12]).

To clarify and further investigate the situation, in this Rapid Communication we present the results of computer simulations of an Ising model containing *strong* bond dilution. Here we restrict our attention to zero-temperature.

The Hamiltonian of the model we work with is given by

$$H=-\sum_{\langle ij\rangle} J_{ij}S_iS_j, \tag{4}$$

where $S_i=\pm 1$ are Ising spins situated on every site of a square $L\times L(=N)$ lattice with periodic boundary conditions and the summation runs over all nearest-neighbor pairs only. The quenched ferromagnetic exchange interactions are selected from a binary distribution given by

$$P(J_{ij})=(1-p)\,\delta(J_{ij})+p\,\delta(J_{ij}-1), \tag{5}$$

where p is the concentration of bonds.

We obtained data for $L=500$ and 750 at zero temperature for a broad range of bond-concentrations ($0\leqslant p\leqslant 0.5$) on a suite of Silicon Graphics workstations and for $L=1000$ on a SGI Origin 2000; as the data for the different lattice sizes studied are practically indistinguishable, here we simply present the results for the largest lattice simulated. We begin each run with a random starting configuration of the spins and then update the lattice by first calculating the energy change that would result from flipping a spin. The rule we

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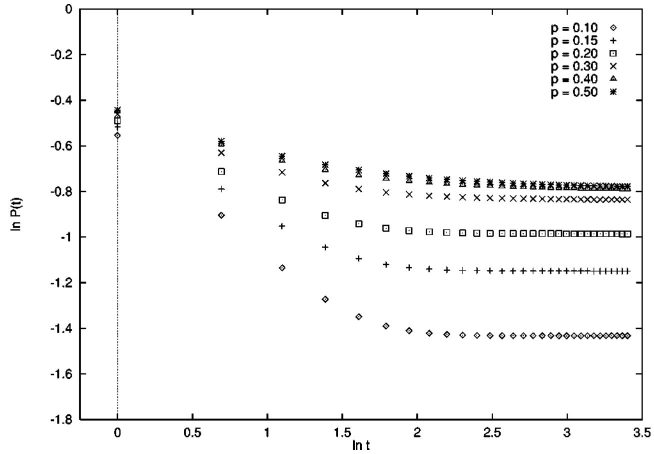

FIG. 1. Log-log plot of $P(t)$ vs t for the bond-diluted 2D Ising model for a range of bond concentrations p; the size of the lattice is 1000×1000.

use is, always flip if the energy change is negative, never flip if the energy change is positive, and flip at random if the energy change is zero.

The number, $n(t)$, of spins that have never flipped until time t is then counted. As we are working with strongly diluted lattices, it is necessary to monitor the value of $n(t)$ after practically each Monte Carlo step. The persistence probability is given by [1]

$$P(t)=[\langle n(t)\rangle]/N, \tag{6}$$

where $\langle\cdots\rangle$ indicates an average very different initial conditions and $[\cdots]$ denotes an average over samples, i.e., over the bond-dilution. For the simulations considered in this work we averaged over at least 100 different initial conditions and samples for each run.

We now discuss our results. To examine the decay of the persistence probability, in Fig. 1 we plot $\ln P(t)$ versus $\ln t$ for a wide range of bond concentrations, p: $0.1\leqslant p\leqslant 0.5$, for a lattice of size $L=1000$. The decay of $P(t)$ appears to be nonalgebraic before ''freezing'' occurs. We see that, effectively, $P(t)=P(\infty)$ for $t>t^*(p)$,where the value $t^*(p)$ depends on the strength of the dilution. Furthermore, the nonzero value of $P(\infty)$ also depends on p, with the fraction of nonflipping spins increasing monotonically with the bond concentrations. The increase in $P(\infty)$ with p can be seen more clearly in Fig. 2, where we have plotted some additional data at values of the exchange interaction not shown in Fig. 1. The numerical values of $P(\infty)$ for the different bond concentration simulated are also displayed in Table I.

TABLE I. The fraction of spins, $P(\infty)$, which *never* flip at various values of the bond concentration, p.

p	$P(\infty)$
0	0
0.025	0.0708(1)
0.050	0.1340(1)
0.075	0.1900(1)
0.100	0.2390(1)
0.125	0.2813(1)
0.150	0.3173(1)
0.175	0.3479(2)
0.200	0.3732(2)
0.250	0.4097(1)
0.300	0.4331(2)
0.350	0.4453(3)
0.400	0.4526(3)
0.450	0.4559(2)
0.500	0.4576(1)

Obviously, when $p=0$ all spins eventually flip as the energy change in flipping is always zero. For a value of $p\neq 0$, there will be regions of the lattice containing finite clusters where it will cost in energy to flip spins. For example, an isolated bond connecting two up spins is just such a stable cluster. The occurrence of these clusters increases with the bond concentration and hence also does the fraction of spins which never flip. This increases smoothly to $p=0.5$, the bond percolation threshold, where it appears to level off. That is, the maximum value of $P(\infty)\sim 0.46$. Clearly, $P(\infty)$ must decrease eventually for higher values of p, as we know that every spin flips infinitely many times for the pure model, $p=1$ [1,11].

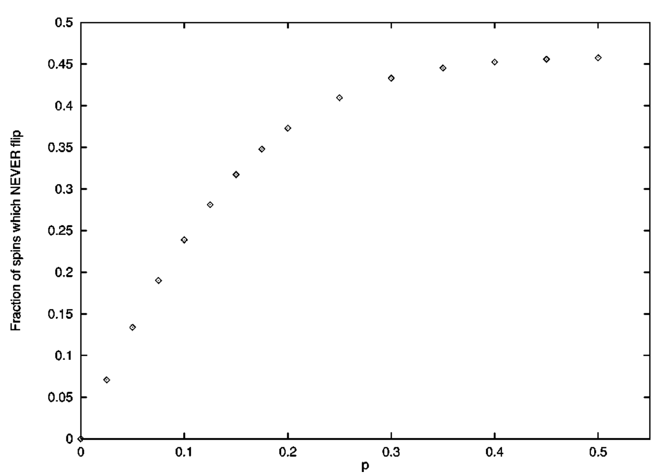

FIG. 2. A plot of the fraction of spin which *never* flip $[P(\infty)]$ against the bond concentration p.

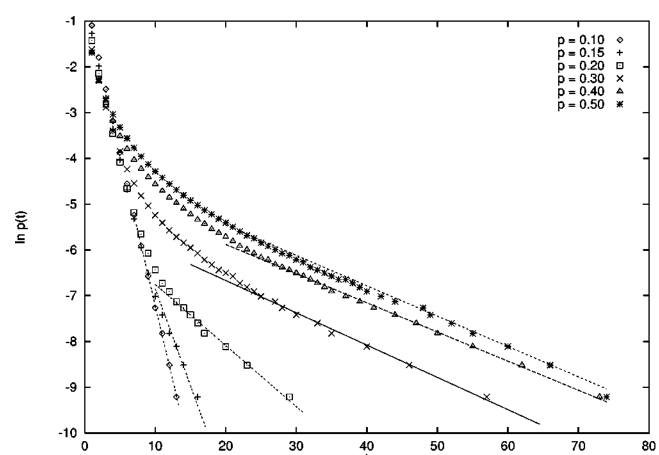

FIG. 3. Plot of $\ln p(t)$ against t for different bond concentrations p. The straight lines are linear fits to the data after discarding the initial short time behavior.

We now consider the nonalgebraic decay of $P(t)$ to $P(\infty)$. As discussed earlier, it is more convenient to work with $p(t)$ from Eq. (2). In Fig. 3 we replot the data displayed in Fig. 1 as $\ln p(t)$ against t. The straight lines are linear fits to Eq. (3) after discarding data for short times. It is evident from Fig. 3 that $p(t)$ indeed decays exponentially at large times. Hence, we confirm that for the strongly diluted Ising model in 2D persistence decays exponentially as predicted by Newman and Stein [11]. This is in marked contrast to the behavior for the pure [1–3] and the weakly diluted models [10].

To conclude, we have presented data for the zero-temperature dynamics of the strongly diluted random-bond 2D Ising ferromagnet. This system exhibits ''blocking'' and we find evidence that $p(t)$ decreases exponentially for large times. The fraction of spins that *never* flip increases monotonically for zero with increasing bond concentration. Our results support the suggestion that the decay of the persistence probability can be nonalgebraic for certain classes of models. Indeed, for the diluted 2D Ising model the behavior of $p(t)$ would appear to depend crucially on the strength of the dilution.

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